Part XII

Proper Time and Physical Clock Measurement

Franc Rozman

Abstract

A physical atomic clock measures time locally by accumulating cycles of a defined reference process. For clock i, the measured interval is

$$\Delta t_i = \frac{\Delta N_i}{N_0},$$

where ΔN_i is the accumulated number of reference cycles and N_0 defines the second. Relativistic proper time is defined as the time measured by that same local clock. Consequently, for the same clock and the same pair of local events,

$$\boxed{\Delta \tau_i = \Delta t_i = \frac{\Delta N_i}{N_0}.}$$

This paper examines the physical implications of this identity. A clock produces one local physical record; it does not independently measure a local time t_i and a proper time τ_i. Different reference systems may describe the same clock using different coordinate-time intervals and different relative velocities, but every valid relativistic reconstruction of the same clock interval must reproduce the unique locally measured proper time. Thus, coordinate description and physical clock accumulation must be clearly distinguished.

Relativistic clock predictions are therefore ultimately experimentally testable as differences between physical clock records, rather than merely as differences between coordinate descriptions. Space-based atomic-clock experiments such as ACES provide a particularly direct setting for comparing these predictions with locally accumulated clock intervals.

1. Introduction

Time dilation is one of the fundamental predictions of relativity. It is commonly formulated by distinguishing coordinate time from proper time and by relating these quantities through the relative motion of clocks, their trajectories, and, in general relativity, the gravitational field.

A physical clock, however, performs a much simpler operation. It does not measure relative velocity, a Lorentz transformation, or a coordinate system. It locally accumulates cycles of a physical reference process. For an ideal atomic clock i, the measured time interval can therefore be expressed as

$$\boxed{\Delta t_i = \frac{\Delta N_i}{N_0}\ \mathrm{s},}$$

where ΔNi is the number of reference cycles accumulated by the clock and N0 is the number of such cycles defining one second.

Throughout this paper, Δti denotes this **locally measured clock interval**. It should not be confused with the coordinate time conventionally denoted by t in relativistic equations. Coordinate time will be introduced separately when required.

Relativity defines the proper-time interval Δτi as the time interval measured locally by a clock along its own history. For the same clock and the same pair of local events, the locally measured interval and the proper-time interval therefore refer to the same physical record:

$$\boxed{\Delta \tau_i = \Delta t_i = \frac{\Delta N_i}{N_0}\ \mathrm{s}.}$$

This elementary identity motivates the central question of this paper. If both Δti and Δτi refer to the interval measured by the same physical clock, what physical distinction remains between them?

The question becomes particularly important when a single clock is described from several different reference systems. Clock A, for example, may have different relative velocities with respect to observers B,C,D,.... There is therefore no unique relative velocity v_A that can characterize the physical operation of clock A. Nevertheless, between two specified events on its history, clock A produces only one physical record,

$$\Delta N_A,$$

and consequently one locally measured interval,

$$\Delta t_A,$$

and one proper-time interval,

$$\Delta \tau_A.$$

All valid relativistic descriptions of this same interval must therefore reproduce

$$\boxed{\Delta \tau_A = \Delta t_A = \frac{\Delta N_A}{N_0},}$$

independently of the reference system from which the calculation is performed.

This observation separates two operations that are often discussed together. The first is the **physical measurement of time** by a clock through the accumulation of cycles. The second is the **mathematical description or prediction of that clock interval** from a selected reference system. Different coordinate systems may assign different coordinate times, velocities, and spatial coordinates to the same clock history, but they cannot change the physical cycle count already accumulated by the clock.

The distinction becomes experimentally decisive when two independent clocks are compared. Each clock produces its own local record,

$$\Delta N_A \quad \text{and} \quad \Delta N_B.$$

If the records are equal,

$$\Delta N_A = \Delta N_B,$$

then necessarily

$$\Delta t_A = \Delta t_B$$

and

$$\Delta \tau_A = \Delta \tau_B.$$

If the records differ, then the difference represents a real difference in locally accumulated clock intervals and requires a corresponding physical and theoretical explanation. It cannot be created or removed merely by changing the reference system used to describe the clocks.

The purpose of this paper is therefore not to question the mathematical consistency of Lorentz transformations as coordinate transformations. It is to examine more specifically the relation between three quantities associated with a physical clock:

$$\boxed{\Delta N_i, \qquad \Delta t_i, \qquad \Delta \tau_i.}$$

The analysis asks what is directly measured, what is calculated from a relativistic description, and what consistency conditions must hold between the two. In particular, it examines the requirement that every valid description of a given clock interval reproduce the same locally accumulated physical record.

The resulting proposition is experimentally testable. Any predicted difference between the proper times of two atomic clocks must ultimately appear as a difference between their accumulated physical clock records. Precision atomic-clock comparisons therefore provide a direct empirical test of the relation developed in this paper. Precision atomic-clock comparisons provide a direct means of answering this question, while relativistic calculations provide predictions that must ultimately be tested against those physical clock records.

2. Measurement of Time

A physical clock measures a time interval locally by accumulating repeated cycles of a reference process. The measurement is associated with two events occurring at the clock itself.

Let D_1 and D_2 denote two such local events for an ideal atomic clock i, and let

$$\Delta N_i = N_i(D_2) - N_i(D_1)$$

be the number of reference cycles accumulated between them.

The SI second provides the unit in which this interval is expressed. For the cesium-133 reference transition, one second corresponds to

$$N_0 = 9\,192\,631\,770$$

periods of the specified radiation. The locally measured time interval is therefore

$$\boxed{\Delta t_i = \frac{\Delta N_i}{N_0}\ \mathrm{s}.}$$

2.1 Meaning of Δt_i

Throughout this paper, the symbol

$$\boxed{\Delta t_i}$$

denotes the **time interval locally measured by clock i**.

This notation is deliberately distinguished from coordinate time used in relativistic calculations. In standard relativistic notation the symbol t is commonly used for coordinate time. To avoid ambiguity, when a coordinate-time interval is required later in this paper, it will be denoted by

$$\boxed{\Delta T_j,}$$

where j identifies the reference system in which that coordinate interval is defined.

Thus,

$$\Delta t_A$$

means the interval physically measured by clock A, whereas

$$\Delta T_B$$

means a coordinate-time interval defined in reference system B.

These quantities must not be confused.

2.2 The Physical Clock Record

The primary physical record produced by clock i is the accumulated cycle count

$$\Delta N_i.$$

The corresponding locally measured interval is obtained directly from

$$\Delta t_i = \frac{\Delta N_i}{N_0}.$$

The clock does not need to know its velocity relative to another observer, its position in an external coordinate system, or the trajectory by which it arrived at its present location in order to produce this local record.

Its measurement is therefore autonomous:

$$\boxed{\Delta N_i \longrightarrow \Delta t_i.}$$

For two ideal clocks A and B,

$$\Delta t_A = \frac{\Delta N_A}{N_0},$$

and

$$\Delta t_B = \frac{\Delta N_B}{N_0}.$$

Both measurements are expressed using the same defined unit of time.

2.3 Equal Seconds and Equal Intervals

The equality of the unit must be distinguished from the equality of accumulated intervals.

Both ideal clocks realize the same defined second:

$$\boxed{1\,\mathrm{s}_A = 1\,\mathrm{s}_B.}$$

Equivalently, the same number N0 of reference periods corresponds to one second for either clock.

If

$$\Delta N_A = \Delta N_B,$$

then necessarily

$$\boxed{\Delta t_A = \Delta t_B.}$$

If instead

$$\Delta N_A \neq \Delta N_B,$$

then the measured intervals are different:

$$\boxed{\Delta t_A \neq \Delta t_B.}$$

The distinction is important because it keeps the definition of the unit separate from the physical result of a clock comparison.

2.4 What Is and Is Not Part of the Measurement

The local measurement relation

$$\boxed{\Delta t_i = \frac{\Delta N_i}{N_0}}$$

contains no relative velocity and no coordinate transformation.

Quantities such as

$$v_{A/B}, \qquad v_{A/C}, \qquad \Delta T_B, \qquad \Delta T_C$$

may later be introduced to describe the clock from different reference systems. They are not additional quantities measured by clock A when it determines its own local interval.

This distinction establishes the terminology used throughout the paper:

$$\boxed{\Delta N_i = \text{physical cycle count},}$$

$$\boxed{\Delta t_i = \text{locally measured clock interval},}$$

and, when required,

ΔT_j = coordinate-time interval in reference system j.

The next question is then whether relativistic proper time introduces a physically distinct clock quantity or whether, for the same clock and the same local events, it refers to the same measured interval.

3. Proper Time

Relativity defines **proper time** as the time measured by a clock along its own history. Consider an ideal clock A and two local events A1 and A2 occurring at that clock. The proper-time interval between these events is denoted by

$$\Delta\tau_A.$$

In the terminology introduced in the preceding section, the interval locally measured by the same clock between the same two events is

$$\Delta t_A = \frac{\Delta N_A}{N_0}\ \mathrm{s}.$$

Since proper time is defined as the interval measured by that clock, the two quantities refer to the same physical clock record:

$$\boxed{\Delta\tau_A = \Delta t_A = \frac{\Delta N_A}{N_0}\ \mathrm{s}.}$$

The same relation holds for every ideal clock i:

$$\boxed{\Delta\tau_i = \Delta t_i.}$$

3.1 One Physical Record

The equality

$$\Delta\tau_i = \Delta t_i$$

does not represent a comparison between two independent measurements. Clock i does not separately record Δti and Δτi. It produces one physical record,

$$\Delta N_i,$$

from which its locally measured interval is obtained.

Thus,

$$\boxed{\Delta N_i \quad \longrightarrow \quad \Delta t_i = \Delta\tau_i.}$$

At the level of local measurement, proper time therefore introduces no second physical clock reading in addition to Δti.

This point is important for the terminology of the present paper. The quantities

$$\Delta t_i$$

And

$$\Delta\tau_i$$

have different conceptual origins: Δti denotes the locally measured clock interval as defined in Section 2, whereas Δτi is the relativistic term for the proper-time interval of that clock. For the same clock and the same two local events, however, their numerical values must be identical.

3.2 Proper Time and Coordinate Time

Proper time must be distinguished from **coordinate time**.

As defined in Section 2, a coordinate-time interval in reference system j will be denoted in this paper by

$$\Delta T_j.$$

In general,

$$\boxed{\Delta T_j \neq \Delta\tau_i}$$

may hold.

This does not imply that clock i possesses two different local times. It means that ΔTj and Δτi describe different quantities: the former is a coordinate interval assigned within reference system j, while the latter is the interval locally recorded by clock i.

The essential distinction is therefore

$$\boxed{\Delta T_j = \text{coordinate-time interval,}}$$

whereas

$$\boxed{\Delta\tau_i = \Delta t_i = \text{locally measured clock interval.}}$$

This separation will be maintained throughout the paper.

3.3 Several Clocks

For several ideal clocks A,B,C,... each clock has its own local physical record:

$$\Delta\tau_A = \Delta t_A = \frac{\Delta N_A}{N_0},$$
$$\Delta\tau_B = \Delta t_B = \frac{\Delta N_B}{N_0},$$
$$\Delta\tau_C = \Delta t_C = \frac{\Delta N_C}{N_0},$$

and generally,

$$\boxed{\Delta\tau_i = \Delta t_i = \frac{\Delta N_i}{N_0}.}$$

These relations establish the equality between proper time and locally measured time **for each clock individually**. They do not yet establish that different clocks have accumulated equal intervals:

$$\Delta t_A \stackrel{?}{=} \Delta t_B.$$

Equivalently, they do not yet establish

$$\Delta N_A \stackrel{?}{=} \Delta N_B.$$

That is a separate physical question.

3.4 The Question Raised by Proper Time

We can now state the conceptual issue precisely.

For a single physical clock,

$$\boxed{\Delta\tau_i = \Delta t_i = \frac{\Delta N_i}{N_0}.}$$

Proper time and locally measured time therefore cannot represent two different physical readings of that clock.

This raises two related questions.

First, if a distant reference system uses coordinate quantities to calculate the proper-time interval of clock i, must that calculation reproduce the locally measured value

$$\Delta t_i?$$

Since proper time is defined by the local clock, the required consistency condition is

$$\boxed{\Delta\tau_i^{(j)} = \Delta t_i}$$

for every valid reconstruction of the same clock interval from reference system j.

Second, when two independent clocks are considered, what determines whether

$$\Delta N_A = \Delta N_B$$

or

$$\Delta N_A \neq \Delta N_B?$$

These are different questions. The first concerns different descriptions of the **same physical clock record**. The second concerns a comparison of **two physical clock records**.

Keeping these two problems, separate is essential for interpreting relativistic clock comparisons.

4. Local Clock Accumulation

An ideal atomic clock measures time through the local accumulation of cycles of a defined reference process. For every clock i,

$$\boxed{\Delta t_i = \frac{\Delta N_i}{N_0},}$$

and, as established in the preceding section,

$$\boxed{\Delta\tau_i = \Delta t_i.}$$

Consequently,

$$\boxed{\Delta\tau_i = \Delta t_i = \frac{\Delta N_i}{N_0}.}$$

These three quantities are not independent descriptions of three different physical processes. They refer to one local clock record.

4.1 Identical Local Clocks

Consider two identical ideal atomic clocks A and B.

Clock A locally realizes the defined atomic reference process, and clock B locally realizes the same reference process. Their measurements are

$$\Delta t_A = \frac{\Delta N_A}{N_0},$$

and

$$\Delta t_B = \frac{\Delta N_B}{N_0}.$$

The definition of the second is the same for both clocks:

$$\boxed{N_0 = 9\,192\,631\,770}$$

reference periods per second.

Thus, one locally realized second at A and one locally realized second at B correspond to the same defined number of reference periods:

$$\boxed{1\,\mathrm{s}_A = 1\,\mathrm{s}_B.}$$

No relative velocity appears in this local definition.

4.2 Local Clock Hypothesis

The present analysis adopts the following physical hypothesis:

> Identical ideal atomic clocks whose local reference processes are subject to identical local physical conditions realize the same local rate of clock accumulation.

A clock does not require knowledge of its velocity relative to surrounding objects in order to realize its atomic reference process. Clock A, for example, may simultaneously have different relative velocities

$$v_{A/B}, \qquad v_{A/C}, \qquad v_{A/D}, \ldots$$

with respect to different reference systems.

There is therefore no unique relative velocity v_A that can be regarded as a local physical property of clock A.

Its local physical record remains

$$\boxed{\Delta N_A \longrightarrow \Delta t_A = \Delta\tau_A.}$$

The same applies independently to clock B.

4.3 Relative Motion and the Local Atomic Process

Relative velocity describes a relation between two systems. It is not, by itself, a locally unique property of either system.

If clock A has velocity

$$v_{A/B}$$

relative to B, and simultaneously

$$v_{A/C}$$

relative to C, with

$$v_{A/B} \neq v_{A/C},$$

the atomic process inside clock A cannot possess two different local rates corresponding to these two external descriptions.

Clock A produces only one sequence of reference cycles,

$$N_A.$$

Therefore it has one locally measured interval

$$\Delta t_A$$

and one proper-time interval

$$\Delta\tau_A.$$

Accordingly,

$$\boxed{\Delta\tau_A = \Delta t_A}$$

is independent of which external reference system is later used to describe clock A.

This does not prohibit different coordinate descriptions. It requires only that every valid description remain consistent with the unique local clock record.

4.4 Equal Local Rates and Accumulated Intervals

For identical ideal clocks under identical local physical conditions, the hypothesis of equal local clock rates can be written as

$$\boxed{\frac{dN_A}{dt_A} = \frac{dN_B}{dt_B} = N_0\ \mathrm{s}^{-1}.}$$

Equivalently, each locally measured second corresponds to the same number N_0 accumulated reference cycles.

If equal local intervals are considered,

$$\Delta t_A = \Delta t_B,$$

then necessarily

$$\Delta N_A = \Delta N_B,$$

because

$$\Delta N_A = N_0 \Delta t_A$$

and

$$\Delta N_B = N_0 \Delta t_B.$$

Since

$$\Delta\tau_A = \Delta t_A$$

and

$$\Delta\tau_B = \Delta t_B,$$

it follows directly that

$$\boxed{\Delta t_A = \Delta t_B \quad \Longrightarrow \quad \Delta\tau_A = \Delta\tau_B}$$

and

$$\boxed{\Delta N_A = \Delta N_B.}$$

The equality is therefore a statement about physical clock accumulation, not about how one clock appears from another reference system.

4.5 Consequence for Proper Time

For every individual clock,

$$\boxed{\Delta\tau_i = \Delta t_i = \frac{\Delta N_i}{N_0}.}$$

Therefore a relativistic description of clock A from another reference system cannot assign a second physical proper-time interval to the same clock between the same local events.

It must reconstruct the interval already represented by

$$\Delta N_A.$$

For the same clock interval,

$$\boxed{\Delta\tau_A^{(B)} = \Delta\tau_A^{(C)} = \cdots = \Delta\tau_A = \Delta t_A.}$$

Different reference systems may use different coordinate times and different relative velocities to obtain this result. The physical result itself remains unique.

This establishes the principle used in the following section:

The physical clock record is local and unique different reference systems provide different descriptions of it.

Any valid relativistic description must preserve this distinction,

5. Relativistic Description and Physical Measurement

The preceding sections distinguish the local physical record of a clock from the coordinate description used to represent that clock from another reference system.

For clock A, the locally accumulated number of reference cycles determines

$$\boxed{\Delta t_A = \Delta \tau_A = \frac{\Delta N_A}{N_0}.}$$

This is the physical record of clock A.

Relativity permits the same clock interval to be described from many different reference systems. These descriptions may involve different coordinate times and different relative velocities. However, if they refer to the same clock A and the same two local events, they must all reproduce the same proper-time interval.

This relation is illustrated in Fig. 1.

Figure 1. Local clock record and its reconstruction from different reference systems.

Clock A produces a unique local physical record ΔN, corresponding to the locally measured interval ΔtA and proper-time interval ΔτA. Reference systems B and C may describe the same interval using different coordinate times and relative velocities. A consistent relativistic reconstruction must nevertheless reproduce the same invariant proper-time interval.

5.1 One Clock, Different Reference Systems

Consider clock A described from reference systems B and C.

The relative velocities need not be equal:

$$v_{A/B} \neq v_{A/C}.$$

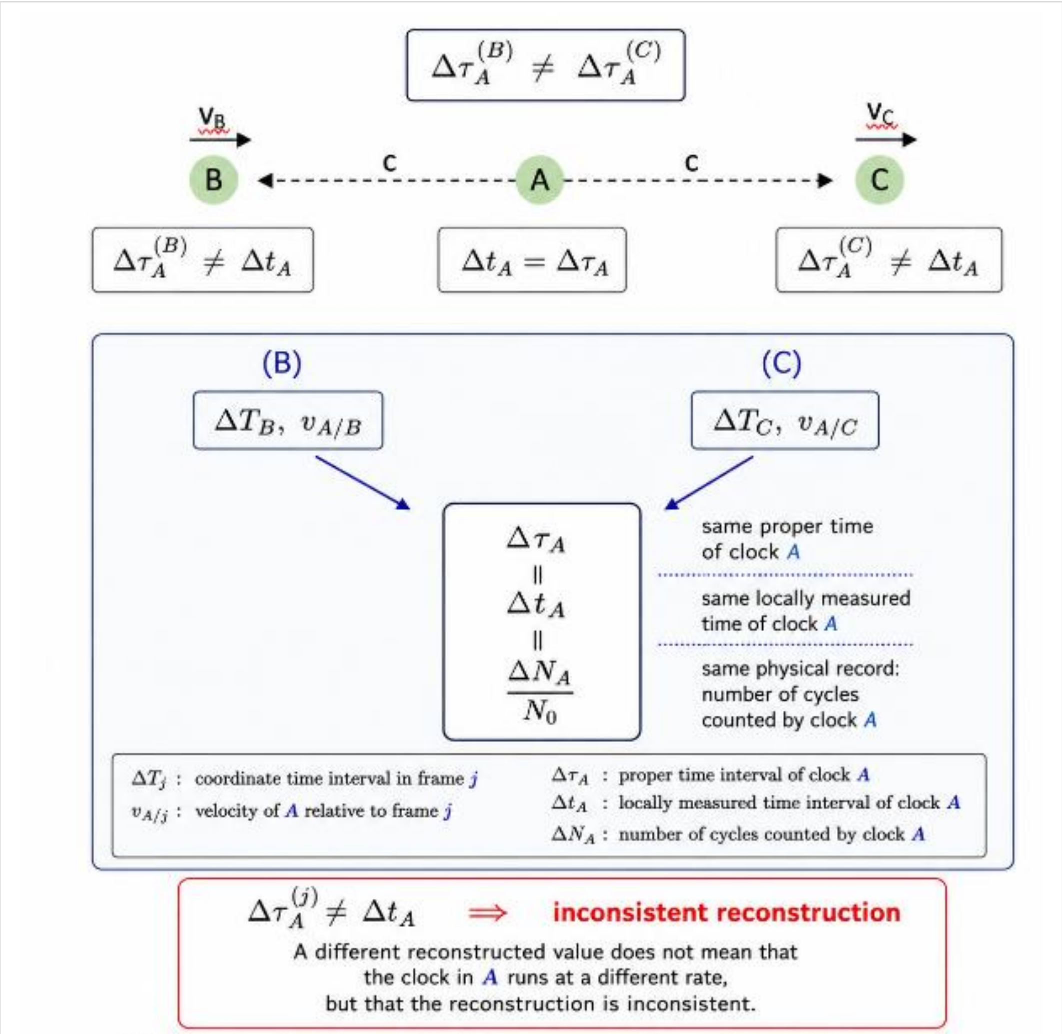

Figure 1. Local clock record and its reconstruction from different reference systems.

Similarly, the coordinate-time intervals assigned in the two systems need not be equal:

$$\Delta T_B \neq \Delta T_C.$$

For simplicity, first consider uniform relative motion. In reference system B, the relativistic relation is

$$d\tau_A = dT_B \sqrt{1 - \frac{v_{A/B}^2}{c^2}}.$$

For a constant relative velocity,

$$\boxed{\Delta\tau_A = \Delta T_B \sqrt{1 - \frac{v_{A/B}^2}{c^2}}.}$$

Reference system C describes the same clock interval as

$$\boxed{\Delta\tau_A = \Delta T_C \sqrt{1 - \frac{v_{A/C}^2}{c^2}}.}$$

Although

$$v_{A/B} \neq v_{A/C}$$

and

$$\Delta T_B \neq \Delta T_C,$$

both calculations must reproduce the same invariant proper time:

$$\boxed{\Delta T_B \sqrt{1 - \frac{v_{A/B}^2}{c^2}} = \Delta T_C \sqrt{1 - \frac{v_{A/C}^2}{c^2}} = \Delta\tau_A.}$$

Since the proper time of clock A is its locally measured interval,

$$\boxed{\Delta T_B \sqrt{1 - \frac{v_{A/B}^2}{c^2}} = \Delta T_C \sqrt{1 - \frac{v_{A/C}^2}{c^2}} = \Delta\tau_A = \Delta t_A = \frac{\Delta N_A}{N_0}.}$$

This equation summarizes the central consistency requirement of the relativistic description.

5.2 A Numerical Example

A simple numerical example makes the distinction transparent.

Suppose clock A locally records

$$\Delta t_A = 10 \text{ s}.$$

Therefore,

$$\Delta\tau_A = 10 \text{ s}.$$

Let reference system B move relative to A with

$$v_{A/B} = 0.6c.$$

The Lorentz factor is

$$\gamma_B = \frac{1}{\sqrt{1 - 0.6^2}} = 1.25.$$

The corresponding coordinate-time interval in system B is therefore

$$\Delta T_B = \gamma_B \Delta\tau_A = 12.5 \text{ s}.$$

When B reconstructs the proper time of clock A,

$$\Delta\tau_A = 12.5\sqrt{1 - 0.6^2},$$

and obtains

$$\boxed{\Delta\tau_A = 10 \text{ s}.}$$

Now let reference system C have a different relative velocity,

$$v_{A/C} = 0.8c.$$

Then

$$\gamma_C = \frac{1}{\sqrt{1 - 0.8^2}} = \frac{5}{3}.$$

The corresponding coordinate interval is

$$\Delta T_C = \gamma_C \Delta\tau_A = 16.667 \text{ s}.$$

The reconstruction gives

$$\Delta\tau_A = 16.667\sqrt{1 - 0.8^2} \approx 10 \text{ s}.$$

Thus,

$$\boxed{\Delta T_B = 12.5 \text{ s}, \qquad \Delta T_C = 16.667 \text{ s},}$$

while

$$\boxed{\Delta\tau_A^{(B)} = \Delta\tau_A^{(C)} = \Delta t_A = 10 \text{ s}.}$$

The coordinate descriptions are different. The physical clock interval is not.

5.3 General Form

The same principle applies when the relative velocity is not constant.

From reference system j,

$$\boxed{\Delta\tau_A = \int_{A_1}^{A_2} \sqrt{1 - \frac{v_{A/j}^2(T_j)}{c^2}}\, dT_j.}$$

Another reference system k may use a different velocity function and a different coordinate time:

$$\boxed{\Delta\tau_A = \int_{A_1}^{A_2} \sqrt{1 - \frac{v_{A/k}^2(T_k)}{c^2}}\, dT_k.}$$

If both calculations describe the same events A_1, A_2 on the same clock A, then

$$\boxed{\int_{A_1}^{A_2} \sqrt{1 - \frac{v_{A/j}^2}{c^2}}\, dT_j = \int_{A_1}^{A_2} \sqrt{1 - \frac{v_{A/k}^2}{c^2}}\, dT_k = \Delta\tau_A.}$$

And because

$$\Delta\tau_A = \Delta t_A,$$

both descriptions must reproduce the locally accumulated cycle count:

$$\boxed{N_0 \int_{A_1}^{A_2} \sqrt{1 - \frac{v_{A/j}^2}{c^2}}\, dT_j = \Delta N_A.}$$

The accumulated number of cycles therefore provides a direct physical consistency condition for the coordinate calculation.

5.4 Coordinate Description Does Not Create a Second Clock Rate

The preceding equations reveal an important distinction.

Clock A itself does not use

$$v_{A/B}$$

or

$$v_{A/C}$$

to determine its local interval. It simply accumulates its physical reference cycles:

$$\Delta N_A.$$

The quantities

$$v_{A/B}, \qquad \Delta T_B$$

and

$$v_{A/C}, \qquad \Delta T_C$$

belong to different coordinate descriptions of this same clock record.

Consequently, the fact that

$$v_{A/B} \neq v_{A/C}$$

cannot imply that clock A simultaneously possesses two different physical rates.

The distinction can be summarized as

different coordinate descriptions are not different physical clock records.

Instead,

$$\boxed{(\Delta T_B, v_{A/B}) \longrightarrow \Delta\tau_A,}$$

and

$$\boxed{(\Delta T_C, v_{A/C}) \longrightarrow \Delta\tau_A,}$$

with

$$\boxed{\Delta\tau_A = \Delta t_A = \frac{\Delta N_A}{N_0}.}$$

Relative velocity is therefore a quantity entering the coordinate reconstruction of the clock interval. It does not provide an additional local reading of clock A.

5.5 Consistency with Physical Measurement

The local cycle count provides a simple test of every reconstruction of clock A.

If clock A records

$$\Delta N_A,$$

then its measured interval is fixed:

$$\Delta t_A = \frac{\Delta N_A}{N_0}$$

Its proper-time interval is consequently

$$\boxed{\Delta \tau_A = \frac{\Delta N_A}{N_0}.}$$

Any calculation from another reference system that claims to reconstruct the proper time of this same clock between the same events must yield this value.

Therefore,

$$\boxed{\Delta \tau_A^{(j)} = \Delta t_A}$$

must hold for every valid reference system j.

If instead a reconstruction gives

$$\Delta \tau_A^{(j)} \neq \Delta t_A,$$

the two values cannot both represent the proper-time interval of clock A between the same events.

The discrepancy cannot be interpreted as clock A possessing a different physical rate merely because it is described from another reference system. It indicates that the reconstruction does not correspond to the specified local clock interval—whether because different events have been compared, the transformation has been applied incorrectly, or the theoretical model does not reproduce the physical measurement.

Thus,

$$\boxed{\text{same clock} + \text{same local events}}$$

necessarily implies

$$\boxed{\text{same } \Delta N_A, \quad \text{same } \Delta t_A, \quad \text{same } \Delta \tau_A.}$$

A reference-frame transformation may change the coordinate quantities used in the description. It cannot change the physical clock record to which that description refers.

6. Experimental Test

The distinction developed in the preceding sections ultimately concerns a measurable physical quantity. For every atomic clock i,

$$\boxed{\Delta \tau_i = \Delta t_i = \frac{\Delta N_i}{N_0}.}$$

A theoretical prediction of a difference between two clock intervals therefore corresponds to a prediction of a difference between their accumulated physical clock records.

The appropriate experimental test is consequently a comparison of highly stable atomic clocks operating under different physical conditions.

The Atomic Clock Ensemble in Space (ACES) provides an exceptionally important realization of such an experiment.

6.1 From Atomic Timekeeping to ACES

The development of atomic clocks transformed time from a quantity measured by macroscopic periodic systems into one referenced to reproducible atomic transitions.

The SI second is presently defined through the caesium-133 hyperfine transition, assigning the fixed value

$$N_0 = 9\,192\,631\,770$$

periods to one second.

Increasing clock stability and accuracy gradually made it possible not only to maintain time but also to use clocks as instruments for testing fundamental physics.

ACES represents a major step in this development. Its origins extend back more than three decades. The mission was conceived to bring a new generation of atomic clocks into the microgravity environment of space and to compare their time and frequency with high-performance clocks distributed on Earth. Early ESA descriptions already identified the mission as an experiment in both fundamental physics and time-and-frequency metrology.

After a long development programme, ACES was launched to the International Space Station on 21 April 2025. On 25 April 2025 it was installed on the Earth-facing side of ESA's Columbus laboratory.

This history is significant. ACES was not conceived merely as a more accurate clock in space. From its beginning, comparison between clocks under different physical conditions was one of the central purposes of the mission.

6.2 The ACES Clocks

At the heart of ACES are two complementary atomic clocks.

PHARAO is a cold-atom caesium clock developed by CNES. Microgravity allows laser-cooled caesium atoms to interact with the clock's microwave field for longer periods than would be practical in an equivalent compact terrestrial apparatus. The second clock is the Space Hydrogen Maser (SHM), which provides excellent short- and medium-term frequency stability. Their complementary properties are combined to form the ACES time reference.

The importance of PHARAO for the present discussion is particularly clear. It is a caesium atomic clock and therefore realizes time through an atomic reference process closely connected with the physical definition of the second.

ACES consequently places an extremely stable atomic reference in an environment physically different from that of terrestrial reference clocks.

The comparison is then between local clock records produced in space and local clock records produced on Earth.

6.3 Original Scientific Objectives

The scientific objectives of ACES were broad from the beginning.

Early ESA mission documents identified three principal goals:

1. demonstration of a new generation of high-performance atomic clocks in space;
2. precise comparison and synchronization of ultra-stable clocks distributed around the world;
3. fundamental-physics tests, particularly an improved measurement of the gravitational redshift and tests involving fundamental constants and relativity.

General relativity predicts that clocks at different gravitational potentials accumulate different proper-time intervals. ACES was designed to compare the space clock with terrestrial clocks with sufficient precision to test this prediction much more accurately than earlier space-clock experiments. Early ESA documentation targeted a gravitational-redshift measurement with relative uncertainty at the level of a few parts in 10^6.

Thus, at its most fundamental level, ACES concerns precisely the physical quantity discussed in this paper:

$$\boxed{\Delta\tau_{\text{space}} - \Delta\tau_{\text{ground}}.}$$

Because proper time is locally measured by the clocks,

$$\Delta\tau_{\text{space}} = \Delta t_{\text{space}} = \frac{\Delta N_{\text{space}}}{N_0},$$

and

$$\Delta\tau_{\text{ground}} = \Delta t_{\text{ground}} = \frac{\Delta N_{\text{ground}}}{N_0}.$$

Therefore a measured proper-time difference corresponds to a difference between physical clock records.

6.4 ACES as a Test of Clock Accumulation

The perspective developed in this paper suggests a particularly direct interpretation of the ACES experiment.

The fundamental comparison is between

$$\Delta N_{\text{space}}$$

and

$$\Delta N_{\text{ground}}.$$

If the accumulated records differ,

$$\Delta N_{\text{space}} \neq \Delta N_{\text{ground}},$$

then the clocks have physically accumulated different local intervals:

$$\boxed{\Delta t_{\text{space}} \neq \Delta t_{\text{ground}}}$$

and therefore

$$\boxed{\Delta \tau_{\text{space}} \neq \Delta \tau_{\text{ground}}.}$$

If instead the accumulated records are equal,

$$\Delta N_{\text{space}} = \Delta N_{\text{ground}},$$

then

$$\boxed{\Delta t_{\text{space}} = \Delta t_{\text{ground}}}$$

and

$$\boxed{\Delta \tau_{\text{space}} = \Delta \tau_{\text{ground}}.}$$

The experiment therefore has a direct physical interpretation independently of how the result is subsequently represented in a coordinate system.

This does not make the time-transfer system unimportant. ACES necessarily requires sophisticated microwave and optical links to compare clocks separated by hundreds or thousands of kilometres. ESA describes the mission as establishing a network connecting the ACES clocks with highly accurate clocks on Earth.

For the conceptual argument of the present paper, however, the detailed operation of these links is secondary. Their purpose is to enable the comparison; they are not the physical clocks whose accumulated intervals are being tested.

The distinction should therefore remain clear:

$$\boxed{\text{local clock accumulation}}$$

is the physical quantity of interest, whereas

$$\boxed{\text{time and frequency transfer}}$$

provides the means by which separated local records are compared.

6.5 Why ACES Is Particularly Important

ACES is especially significant because several developments that were historically separate are brought together in a single experiment: atomic realization of time, operation of an ultra-stable clock in microgravity, comparison with terrestrial reference clocks, and a high-precision test of relativistic predictions.

ESA describes the present scientific programme as testing gravitational time dilation, searching for variations of fundamental constants, enabling relativistic geodesy, and connecting some of the world's best clocks into a global network.

From the perspective of this paper, its importance can be expressed even more simply.

Relativity makes a prediction about proper time.

Atomic clocks produce physical cycle counts.

ACES brings these two statements into experimental contact:

relativistic prediction ⟷ physical clock accumulation.

The comparison should therefore ultimately be interpretable in terms of the quantities

$$\boxed{\Delta N_{\text{space}} \quad \text{and} \quad \Delta N_{\text{ground}}.}$$

Theoretical corrections required to compare separated clocks are essential to the experimental analysis, but they should remain distinguishable from the local physical records produced by the clocks themselves.

ACES is therefore not merely a test of a mathematical transformation. It is a high-precision physical comparison of atomic clocks operating under different conditions, and as such provides a particularly direct experimental setting in which the relation between measured time, proper time, and physical clock accumulation can be examined.

6.6 Two Possible Experimental Outcomes

The relevance of ACES to the present analysis can be stated in terms of two experimentally distinguishable outcomes.

According to the relativistic description, the clock on the International Space Station is affected by both gravitational and kinematic time dilation. At an orbital altitude of approximately 400 km, the weaker gravitational field tends to make the space clock run faster than a clock on the Earth's surface, whereas the orbital velocity tends to make it run slower.

To first order, the combined fractional rate difference can be written approximately as

$$\frac{\Delta f}{f} \simeq \frac{GM}{c^2}\left(\frac{1}{R_E} - \frac{1}{R_E + h}\right) - \frac{v_{\text{ISS}}^2}{2c^2}$$

For representative ISS parameters, the two contributions are approximately

$$\left(\frac{\Delta f}{f}\right)_{\text{grav}} \approx +4.1 \times 10^{-11},$$

and

$$\left(\frac{\Delta f}{f}\right)_{\text{kin}} \approx -3.3 \times 10^{-10}$$

The resulting net prediction is therefore of order

$$\boxed{\frac{\Delta f}{f} \approx -2.9 \times 10^{-10},}$$

corresponding to an accumulated difference of approximately

$$\boxed{25\ \mu\text{s}/\text{day},}$$

with the ISS clock lagging behind a comparable clock on the Earth's surface. The precise prediction for an actual ACES comparison depends on the orbit, the terrestrial clock location, geopotential, and the detailed relativistic model.

The experiment therefore permits two conceptually distinct outcomes to be considered.

If the relativistic prediction is observed, the accumulated clock records will differ:

$$\boxed{\Delta N_{\text{space}} \neq \Delta N_{\text{ground}}.}$$

The difference will constitute a physical difference between the locally accumulated clock intervals, not merely a different coordinate representation.

Alternatively, if after separation of clock accumulation from signal-transfer effects the clocks remain synchronous,

$$\boxed{\Delta N_{\text{space}} = \Delta N_{\text{ground}},}$$

then

$$\boxed{\Delta t_{\text{space}} = \Delta t_{\text{ground}}}$$

and consequently

$$\boxed{\Delta \tau_{\text{space}} = \Delta \tau_{\text{ground}}.}$$

Such a result would support the local-clock hypothesis developed in this paper and would conflict with the predicted relativistic proper-time difference for the specified clock histories.

Thus, ACES provides an empirical counterpart to the mathematical and logical analysis developed in the preceding sections. The argument of this paper is derived from the definitions of local clock measurement, proper time, and the consistency of different reference-frame descriptions. ACES provides the possibility of testing whether the resulting physical interpretation is reflected in actual atomic-clock accumulation.

7. Conclusions

A physical atomic clock measures time locally by accumulating cycles of a reference process. Its measured interval is

$$\Delta t_i = \frac{\Delta N_i}{N_0},$$

where ΔN_i is the accumulated number of reference cycles and N_0 defines the second.

Relativistic proper time is defined as the time measured by that same local clock. For the same clock and the same pair of local events,

$$\boxed{\Delta\tau_i = \Delta t_i = \frac{\Delta N_i}{N_0}.}$$

The quantities Δt_i and $\Delta\tau_i$ therefore do not represent two different physical readings of the clock. They are two descriptions of the same locally accumulated clock interval.

This distinction becomes important when the same clock is described from different reference systems. Clock A may simultaneously have different relative velocities with respect to B,C,D,… and the corresponding coordinate descriptions may contain different coordinate-time intervals. Nevertheless, the physical record of clock A remains unique.

Consequently,

$$\boxed{\Delta\tau_A^{(B)} = \Delta\tau_A^{(C)} = \cdots = \Delta\tau_A = \Delta t_A = \frac{\Delta N_A}{N_0}.}$$

A change of reference system may change the coordinate description of a clock interval, but it cannot change the interval already recorded by the physical clock.

The analysis therefore establishes a clear separation between **local clock accumulation** and **coordinate description**. Relative velocity belongs to the relation between reference systems. It is not a unique local property of a clock, since the same clock can simultaneously possess many different relative velocities with respect to different observers.

For identical ideal atomic clocks operating under identical local physical conditions, this paper adopts the physical hypothesis that their local reference processes realize the same rate of clock accumulation. A genuine change in the operation of an atomic clock must therefore be distinguished from a change produced solely by describing that clock from another reference system.

This distinction leads to a direct consistency requirement:

same clock + same local events $\Longrightarrow$ same ΔN, Δt, Δτ.

Any relativistic reconstruction of the same clock interval must satisfy this requirement.

For two different physical clocks, a predicted difference in proper time has an equally direct physical meaning. Since

$$\Delta\tau_A = \frac{\Delta N_A}{N_0}$$

and

$$\Delta\tau_B = \frac{\Delta N_B}{N_0},$$

a prediction

$$\Delta\tau_A \neq \Delta\tau_B$$

is necessarily a prediction

$$\boxed{\Delta N_A \neq \Delta N_B.}$$

The distinction is therefore experimentally accessible. A proper-time difference between two clocks cannot remain solely a coordinate statement; it must ultimately appear as a difference between their accumulated physical clock records.

This places atomic-clock experiments in a particularly important position. ACES extends the historical development of atomic timekeeping by comparing an ultra-stable space-based atomic reference with high-performance terrestrial clocks under substantially different physical conditions. The relativistic prediction and the physical clock records are thereby brought into direct experimental comparison.

The essential experimental quantities may be expressed as

$$\boxed{\Delta N_{\text{space}} \qquad \text{and} \qquad \Delta N_{\text{ground}}.}$$

Time- and frequency-transfer systems are required to compare these separated records, but they should be conceptually distinguished from the local clock accumulation itself.

The central conclusion of this paper can therefore be stated simply:

A physical clock has one local record.
Different reference systems may describe tha
t record differently.
but they cannot redefine it.

ACES provides a particularly direct empirical test: a measured space–ground difference in accumulated clock cycles would support the relativistic proper-time prediction, whereas synchronous clock accumulation would provide empirical support for the local-clock hypothesis developed in this paper.

The analysis presented here is primarily mathematical and conceptual. It leads to the hypothesis that identical ideal clocks, in the absence of different local perturbations of their reference processes, should maintain equal local clock accumulation. ACES provides an independent empirical test. A relativistically predicted space–ground clock difference would appear as a measurable difference in accumulated clock records, whereas synchronous accumulation would support the local-clock interpretation developed in this paper.

References

[1] Bureau International des Poids et Mesures (BIPM), *The International System of Units (SI)*, 9th ed., updated 2026, Sèvres, France, 2026. DOI: 10.59161/AUEZ1291.

[2] Bureau International des Poids et Mesures (BIPM), *Mise en pratique for the definition of the second in the SI*, Appendix 2 of the SI Brochure, 9th ed., Sèvres, France.

[3] E. F. Taylor and J. A. Wheeler, *Spacetime Physics: Introduction to Special Relativity*, 2nd ed., W. H. Freeman and Company, New York, 1992.

[4] European Space Agency (ESA), *European Users Guide to Low Gravity Platforms*, ESA SP-1296, European Space Agency, 2006, Section 6.4, "ACES."

[5] European Space Agency (ESA), *Scientific Objectives of ACES*, in *Atomic Watch Working Group Report*, ESA SP-1251, European Space Agency, 2005.

[6] European Space Agency (ESA), "ACES: Atomic Clock Ensemble in Space," European Space Agency, Human and Robotic Exploration